

# Package: xactonomial (via r-universe)

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**Type** Package

**Title** Exact Inference for Real-Valued Functionals of k-Sample  
Multinomial Parameters

**Version** 0.4.0

**Date** 2024-06-18

**URL** <https://github.com/sachsmc/xactonomial>

**BugReports** <https://github.com/sachsmc/xactonomial/issues>

**Description** We consider the k sample multinomial problem where we observe k vectors (possibly of different lengths), each representing an independent sample from a multinomial. For a given function  $\psi$  which takes in the concatenated vector of multinomial probabilities and outputs a real number, we are interested in constructing a confidence interval for  $\psi$ . We use an exact (but computational and stochastic) method to compute a confidence interval and a function for calculation of p values. The details of the method will be in a forthcoming preprint.

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**Encoding** UTF-8

**LazyData** true

**Suggests** knitr, rmarkdown

**VignetteBuilder** knitr

**RoxygenNote** 7.3.1

**Config/rextendr/version** 0.3.1.9000

**Repository** <https://sachsmc.r-universe.dev>

**RemoteUrl** <https://github.com/sachsmc/xactonomial>

**RemoteRef** HEAD

**RemoteSha** 7c0820c1c98edbdeab8ad05952ad21b987c524d1

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calc_probs_rust	<i>calculate multinomial probabilities</i>
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### Description

calculate multinomial probabilities

### Usage

```
calc_probs_rust(sar, logt, logc, d, n, nt)
```

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calc_prob_null	<i>Calculate probability for given parameters</i>
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### Description

Given a set of candidate parameter vectors, check if the null  $\psi \leq \psi_0$  is satisfied, and if so, compute the probability for each element of the sample space

### Usage

```
calc_prob_null(theta_cands, psi, psi0, minus1, SSpacearr, logC, II)
```

**Arguments**

theta_cands	A matrix with samples in the rows and the parameters in the columns
psi	The function of interest mapping parameters to the real line
psi0	The null boundary for testing $\psi \leq \psi_0$
minus1	Either plus or minus 1
SSpacearr	A matrix with the sample space for the given size of the problem
logC	log multinomial coefficient for each element of the sample space
II	logical vector of sample space psi being more extreme than the observed psi

**Value**

A numeric vector of probabilities

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calc_prob_null2	<i>Calculate probability for given parameters</i>
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**Description**

Given a set of candidate parameter vectors, check if the null  $\psi \leq \psi_0$  is satisfied, and if so, compute the probability for each element of the sample space

**Usage**

```
calc_prob_null2(theta_cands, psi, psi0, minus1, SSpacearr, logC, II)
```

**Arguments**

theta_cands	A matrix with samples in the rows and the parameters in the columns
psi	The function of interest mapping parameters to the real line
psi0	The null boundary for testing $\psi \leq \psi_0$
minus1	Either plus or minus 1
SSpacearr	A matrix with the sample space for the given size of the problem
logC	log multinomial coefficient for each element of the sample space
II	logical vector of sample space psi being more extreme than the observed psi

**Value**

A numeric vector of probabilities

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expand_index	<i>Get a matrix of indices for all possible combinations of vectors of lengths</i>
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**Description**

This is basically the same as [expand.grid](#), but faster for integers

**Usage**

```
expand_index(lengths)
```

**Arguments**

lengths	A vector with the lengths of each index to expand
---------	---

**Value**

A matrix with `length(lengths)` columns and `prod(lengths)` rows

**Examples**

```
expand_index(c(2, 3, 4))
## the same as
expand.grid(1:2, 1:3, 1:4)
```

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get_theta_random	<i>Sample from the unit simplex in d dimensions</i>
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**Description**

Sample from the unit simplex in d dimensions

**Usage**

```
get_theta_random(d = 4, nsamp = 75)
```

**Arguments**

d	the dimension
nsamp	the number of samples to take uniformly in the d space

**Value**

The grid over Theta, the parameter space. A matrix with d columns and nsamp rows

**Examples**

```
get_theta_random(3, 10)
```

---

itp_root	<i>Find the root of the function f</i>
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**Description**

This finds the value  $x \in [a, b]$  such that  $f(x) = 0$  using the one-dimensional root finding ITP method (Interpolate Truncate Project). Also see [itp](#).

**Usage**

```
itp_root(
  f,
  a,
  b,
  k1 = 0.1,
  k2 = 2,
  n0 = 1,
  eps = 0.005,
  maxiter = 100,
  fa = NULL,
  fb = NULL,
  verbose = FALSE,
  ...
)
```

**Arguments**

f	The function to find the root of in terms of its first (one-dimensional) argument
a	The lower limit
b	The upper limit
k1	A tuning parameter
k2	Another tuning parameter
n0	Another tuning parameter
eps	Convergence tolerance
maxiter	Maximum number of iterations
fa	The value of f(a), if NULL then will be calculated
fb	The value of f(b), if NULL then will be calculated
verbose	Prints out information during iteration
...	Other arguments passed on to f

**Value**

A numeric vector of length 1, the root at the last iteration

**References**

I. F. D. Oliveira and R. H. C. Takahashi. 2020. An Enhancement of the Bisection Method Average Performance Preserving Minmax Optimality. *ACM Trans. Math. Softw.* 47, 1, Article 5 (March 2021), 24 pages. <https://doi.org/10.1145/3423597>

**Examples**

```
fpoly <- function(x) x^3 - x - 2 ## example from the ITP_method wikipedia entry
itp_root(fpoly, 1, 2, eps = .0001, verbose = TRUE)
```

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log_multinom_coef	<i>Calculate log of multinomial coefficient</i>
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**Description**

Calculate log of multinomial coefficient

**Usage**

```
log_multinom_coef(x, sumx)
```

**Arguments**

x	Vector of observed counts in each cell
size	Total count

**Value**

The log multinomial coefficient

**Examples**

```
## @examples
S0 <- sspace_multinom(4, 6)
S1 <- sspace_multinom(4, 7)
logC0<- apply(S0,1,log_multinom_coef,sumx=6)
logC1<- apply(S1,1,log_multinom_coef,sumx=7)
logC<- outer(logC0,logC1,'+')
```

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pvalue\_psi0

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*Compute a p value for the test of  $\psi \leq \psi_0$* 


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**Description**

Compute a p value for the test of  $\psi \leq \psi_0$

**Usage**

```
pvalue_psi0(
  psi0,
  psi,
  psi_hat,
  psi_obs,
  maxit,
  chunksize,
  lower = TRUE,
  target,
  SSpacearr,
  logC,
  d_k
)
```

**Arguments**

psi0	The null value
psi	The function of interest
psi_hat	The vector of psi values at each element of the sample space
psi_obs	The observed estimate
maxit	Maximum iterations
chunksize	Chunk size
lower	Do a one sided test of the null that it is less than psi0, otherwise greater.
target	Stop the algorithm if $p \geq \text{target}$ (for speed)
SSpacearr	The sample space array
logC	The log multinomial coefficient
d_k	The vector of dimensions

**Value**

A p-value

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rust_sspace	<i>calculate sample space of a multinomial with dimension <math>d</math> and sample size <math>n</math></i>
-------------	---

---

**Description**

calculate sample space of a multinomial with dimension  $d$  and sample size  $n$

**Usage**

```
rust_sspace(d, n)
```

---

sample_unit_simplex	<i>Return a random sample from the <math>d</math> unit simplex</i>
---------------------	--

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**Description**

Return a random sample from the  $d$  unit simplex

**Usage**

```
sample_unit_simplex(d)
```

---

sample_unit_simplexn	<i>Return <math>n</math> random samples from the <math>d</math> unit simplex</i>
----------------------	--

---

**Description**

Return  $n$  random samples from the  $d$  unit simplex

**Usage**

```
sample_unit_simplexn(d, n)
```



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sspace_multinom	<i>Enumerate the sample space of a multinomial</i>
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**Description**

We have  $d$  mutually exclusive outcomes and  $n$  independent trials. This function enumerates all possible vectors of length  $d$  of counts of each outcome for  $n$  trials, i.e., the sample space. The result is output as a matrix with  $d$  columns where each row represents a possible observation.

**Usage**

```
sspace_multinom(d, n)
```

**Arguments**

d	Dimension
n	Size

**Value**

A matrix with  $d$  columns

**Examples**

```
d4s <- sspace_multinom(4, 8)
stopifnot(abs(sum(apply(d4s, 1, dmultinom, prob = rep(.25, 4))) - 1) < 1e-12)
```

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xactnomial	<i>Exact inference for a real-valued function of multinomial parameters</i>
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**Description**

We consider the  $k$  sample multinomial problem where we observe  $k$  vectors (possibly of different lengths), each representing an independent sample from a multinomial. For a given function  $\psi$  which takes in the concatenated vector of multinomial probabilities and outputs a real number, we are interested in constructing a confidence interval for  $\psi$ .

**Usage**

```
xactonomial(
  psi,
  data,
  psi0 = NULL,
  alpha = 0.05,
  psi_limits,
  maxit = 50,
  chunksize = 500,
  conf.int = TRUE
)
```

**Arguments**

psi	Function that takes in a vector of parameters and outputs a real valued number
data	A list with k elements representing the vectors of counts of a k-sample multinomial
psi0	The null hypothesis value. A p value for a test of $\psi \leq \psi_0$ is computed. If NULL only a confidence interval is computed.
alpha	A 1 - alpha percent confidence interval will be computed
psi_limits	A vector of length 2 giving the lower and upper limits of the range of $\psi(\theta)$
maxit	Maximum number of iterations of the stochastic procedure
chunksize	The number of samples taken from the parameter space at each iteration
conf.int	Logical. If FALSE, no confidence interval is calculated, only the p-value.

**Details**

Let  $T_j$  be distributed Multinomial $_{d_j}(\theta_j, n_j)$  for  $j = 1, \dots, k$  and denote  $\mathbf{T} = (T_1, \dots, T_k)$  and  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_k)$ . The subscript  $d_j$  denotes the dimension of the multinomial. Suppose one is interested in the parameter  $\psi(\boldsymbol{\theta}) \in \Psi \subseteq \mathbb{R}$ . Given a sample of size  $n$  from  $\mathbf{T}$ , one can estimate  $\boldsymbol{\theta}$  with the sample proportions as  $\hat{\boldsymbol{\theta}}$  and hence  $\psi(\hat{\boldsymbol{\theta}})$ . This function constructs a  $1 - \alpha$  percent confidence interval for  $\psi(\boldsymbol{\theta})$  and provides a function to calculate a p value for a test of the null hypothesis  $H_0 : \psi(\boldsymbol{\theta}) \leq \psi_0$ . We make no assumptions and do not rely on large sample approximations. The computation is somewhat involved so it is best for small sample sizes.

**Value**

A list with 3 elements: the estimate, the 1 - alpha percent confidence interval, and p-value

**Examples**

```
psi_ba <- function(theta) {
  theta1 <- theta[1:4]
  theta2 <- theta[5:8]
  sum(sqrt(theta1 * theta2))
}
data <- list(T1 = c(2,1,2,1), T2 = c(0,1,3,3))
```

```
xactonomial(psi_ba, data, psi_limits = c(0, 1), maxit = 5, chunksize = 20)
```

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